Homework 6 (Vector Analysis)

1. Prove the Gauss law for finite point charges: Given a compact three surface Ω with charges $q_1, \ldots, q_N$ in it, the electric flux generated by these charges across $\partial\Omega$ is $\sum_{i=1}^N q_i$.

2. Show that in vacuum (i.e. $\rho, J = 0$), **E** and **B** satisfying Maxwell's equations must also satisfy the wave equations

$$\begin{cases} E_{tt} - \Delta E = 0 \\ B_{tt} - \Delta B = 0. \end{cases}$$

3. On Minkowski space $\mathbb{R}^{1,3}$, use the general rule

$$\alpha \wedge *\beta =< \alpha, \beta > dt \wedge dx \wedge dy \wedge dz$$

   to verify the table for star operator $*$ on two forms given in class.

4. Prove that, any closed self-dual or anti-self-dual two form on usual $\mathbb{R}^4$ with *compact support* must be 0. (Recall that support of a form is the closure of the set where the form is nonzero). The "usual" $\mathbb{R}^4$ means that we are taking the ordinary dot product for inner product, not the Minkowski inner product.